The Effect of Quasi-Optics Errors on Reflector Antenna Performance

Patricia R. Foster, *Member, IEEE,* and Richard J. Wylde

Abstract—**The use of quasi-optics feed systems with a reflector antenna allows multiple frequency operation over a very wide bandwidth for remote sensing of the earth and atmosphere from space. The errors in such a multichannel quasi-optical feed system must be controlled in order to limit the degradation of system performance. These errors can be attributed to the presence of higher order Gaussian modes but the errors can be minimised if the quasi-optical system can be made independent of frequency. If volume limitations preclude a frequency-independent design, the amplitude of the higher order Gaussian modes must be controlled.**

I. Introduction

THE DEVELOPMENT of multichannel systems at millimetre wavelengths for remote sensing of the earth and atmosphere from space is leading to the increased use of quasi-optics feed systems for reflector antennas. The limits on allowable variations in the antenna beam efficiency will impose tight limits on how far the performance of the quasi-optics system can stray from the ideal. For example, for METEOSAT MWS [1] and AMSU-B [2], the antenna beam efficiency must be greater than 95% at frequencies between 89 and 192 GHz while the footprints on the earth must be matched to within 10% over the same frequency range. For SEASAT [3], the required beam efficiency was greater than 87% over the frequency range 6.633 to 37.0 GHz. Over such a broad frequency range, no attempt was made to keep the footprint constant.

A system is described as 'quasi-optical' if its cross-section is limited in terms of wavelengths. Any beam passing through the quasi-optics system undergoes diffractive spreading of the beam which must be controlled [4]. In each frequency chain, a multichannel quasi-optical system will contain several optical components which may be mirrors, lenses and dichroic plates. The geometrical arrangement of these may be limited by available space and an ideal optical chain may not be possible.

As far as the reflector antenna is concerned, the quasi-optics output at the focus (primary or secondary) of the reflector is just an input feed pattern and the problem addressed in this paper is to determine how well the quasi-optics output matches the focal field of the reflector.

This reduces to studying two aspects of quasi-optics performance, which are the position of the final beam waist and the shape of the output radiation pattern from the quasi-optics system. The influence of these parameters on the final performance of the associated reflector system and their variation with frequency is discussed in this paper.

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P. R. Foster is with Microwave and Antenna Systems, 16 Peachfield Road, Great Malvern, Worcestershire, WR14 4AP, United Kingdom.

R. J. Wylde is with Thomas Keating Ltd., Precision Engineers, Station Mills, Billinghurst, Sussex, RH14 9SW, United Kingdom.

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II. Parametric Description of Antenna Performance

The parameters describing antenna performance for a remote sensing system are not those of a communications or radar antenna. In remote sensing, the two parameters of prime interest are antenna beam efficiency and beam width. Peak gain and sidelobes are only important in that they affect the antenna beam efficiency (1).

For a power radiation pattern defined as $P(\theta, \phi)$, the antenna beam efficiency, η_B, is defined as the power contained within in a cone of semiangle, θ_0, about the antenna boresight:

$$\eta_B = \frac{\int_0^{2\pi} \int_0^{\theta_0} P(\theta, \phi) \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\pi} P(\theta, \phi) \sin\theta \, d\theta \, d\phi}. \tag{1}$$

The limit of integration, θ_0, may be defined in various ways, such as the first null of the radiation pattern or the angle at which the gain is -20.0 dB with respect to the peak gain. A third common definition, that θ_0 is 1.25 times the footprint which is the beamwidth to halfpower, is adopted in this paper [1], [2]. If the first sidelobe is below -20.0 dB, these three different method of defining the angular limit will provide beam efficiencies within 1% of each other.

The antenna beam efficiency must be as high as possible to avoid using the complex algorithms to remove the effects of adjacent pixels. This implies that the sidelobes must be as low as possible which leads to a large edge taper in the aperture illumination. The requirement for footprint (or beamwidth) equality across a wide bandwidth leads to the use of "under-illumination." If the feed illumination was fundamental mode Gaussian, the output radiation pattern from the reflector would also be Gaussian in shape. Holding the output beam waist of the quasi-optics system constant in linear dimensions at all frequencies would result in a constant footprint with frequency. This holds for 100% of the power in the fundamental mode. Any admixture of higher order modes will degrade this.

For a multichannel system, the Gaussian source at the focal point is the final beamwaist of a quasi-optics system. However, the output at each frequency will be subject to degradation and the illumination of the reflector is only an approximation to the required Gaussian. Sections III to V consider the results of typical errors in a quasi-optics system on the final parameters of a remote sensing reflector system.

III. Gaussian Beam-Mode Analysis

Gaussian beam-mode analysis is well-known [4], [5] and is an appropriate tool to examine these two aspects to first order. The output radiation pattern of any quasi-optical system is determined by the amplitude and relative phase of the Gaussian beam modes present. Associated with each propagating Gaussian beam mode is an on-axis phase slippage relative to a plane wave. Unlike the beamwaist, W, and curvature, R, of the orthonormal set of propagating modes, this on-axis slippage is mode-number dependent. The slippage is caused by the finite width of the propagating beam and is a characteristic of quasi-optical systems. It is analogous to the wavelength in a waveguide exceeding the free space value. The existence of this slippage results in a changing output field even if the amplitude coefficients of the various modes remain unchanged.

A corrugated horn is well described by 10 Gaussian beam-modes with azimuthal symmetry [6]. Given an appropriate choice of beamwidth to aperture radius (W = 0.6435 of the aperture radius), some 98% of the power is in the fundamental mode. Fig. 1 shows the field distribution for the fundamental Gaussian mode and three higher order modes. If such a horn is used as a source radiator in a quasi-optical system, the relative phases of the modes will be altered as the beam passes through the various components of the optical chain. In addition, as the components will be of finite size, some of the higher order modes will suffer truncation and there will be an interchange of modal content which will alter the relative amplitudes of the modes.

Calculations using Wolfram's MATHEMATICA program to test the effect of truncation have been carried out, based upon a corrugated horn mode set suffering a $\pi/2$ fundamental phase shift followed by truncation at some distance off the axis. This phase slippage would be produced at the output focal plane of, say, a mirror illuminated by a corrugated horn of zero flare angle with its aperture (or farfield phase centre) placed at the input focal plane of the mirror. The extent of cross-coupling at a truncating element can be determined from the cross-coupling integrals which include the relative phase of the incoming modes.

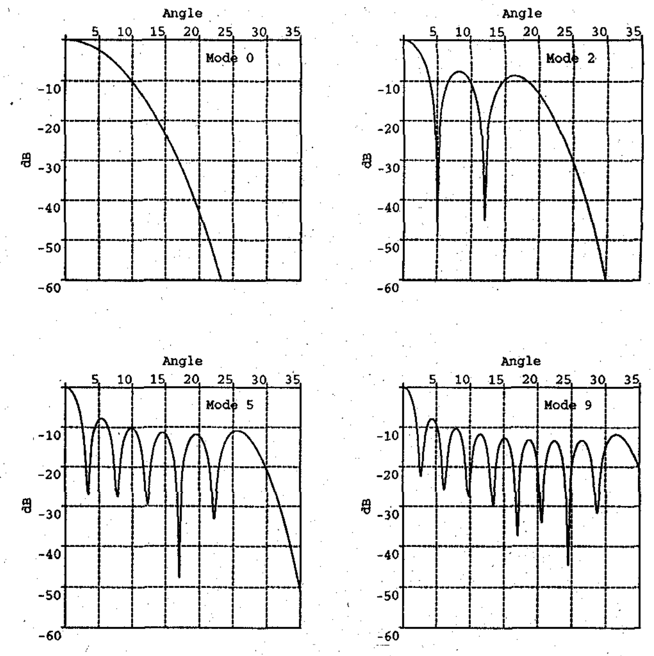

Fig. 1. Farfield radiation patterns for a single Gaussian mode with a beamwaist of 1.91 wavelengths.

The power, η_p, coupled into mode number p, can be expressed in terms of the modal amplitude, A_p, and the truncated field, $E^T(r)$, by way of the Laguerre functions, L_p^0.

$$A_p = \frac{4}{2\pi W^2} \int_0^a E^T(r)\, L_p^0[2r^2/W^2]\, e^{-r^2/W^2} 2\pi r\, dr \quad (2)$$

$$\eta_p = \frac{\int_0^a A_p A_p^* (L_p^0[2r^2/W^2])^2 e^{-r^2/W^2} 2\pi r\, dr}{\int_0^a 2\pi r |E^T(r)|^2\, dr} \quad (3)$$

Equation (3) may be rewritten as

$$\eta_p = \frac{W^2 A_p^2/4}{\int_0^a 2\pi r |E^T(r)|^2\, dr}. \quad (4)$$

In the above equations, a is the radius of the mirror and W is the arbitrary beam width parameter common to the modes. The truncation is equivalent to a circular aperture whose surface is totally absorbing and whose edge is non-diffracting. The field is first truncated and then decomposed to find the new modal coefficients. The choice of W is controlled by the need to maximise the power in the fundamental mode. In the general case, this choice of optimisation will result in the output beamwidth, W_{out}, being different from the input beamwidth, W_{in}.

A single truncation with an aperture radius of 1.8 W_{in} of a corrugated horn mode set at the output focal plane of a mirror increases the power in the fundamental from 98% to 99.3% when $W_{\text{out}} = 1.01\, W_{\text{in}}$. However truncation at 1.2 W_{in} reduces the power in the fundamental to 93.8% (Table I).

TABLE I
MODAL AMPLITUDE FOR A CORRUGATED HORN TRUNCATED AT 1.8 W_{in} AND 1.2 W_{in} WITH THE OUTPUT WAIST W_{out} = 1.01 TIMES THE INPUT WAIST, W_{in}

Mode No.	Horn	Truncated 1.8 W_{in}	Aperture 1.2 W_{in}
Fundamental	0.980700	0.993300	0.938127
1	<0.000001	0.000016	0.019639
2	0.014500	0.005860	0.010455
3	0.001850	0.000112	0.010880
4	0.000385	0.000336	0.001506
5	0.001160	0.000156	0.000392
6	0.000399	0.000050	0.002389
7	<0.000001	0.000049	0.002509
8	0.000150	0.000028	0.001140
9	0.000230	<0.000001	0.000120

Truncation is not necessarily detrimental to the form of a propagating beam and could be used to "clean up" the form of the beam by reducing the amplitudes of higher order modes. Adding additional baffles will cease to have value when the power removed from the second higher order mode (which is dominant) balances the power added to that mode from the fundamental mode. The phase of the modes is important. For a narrow band system with short path lengths, baffles could be added to introduce some power from the fundamental mode into a higher order mode with the correct phase to cancel the existing higher order mode power. In broadband systems, such a solution is not possible because of the difficulty of sustaining the correct modal phasing over a large bandwidth.

IV. Position of the Final Beam Waist

If the final beam waist of the quasi-optics system is constant in dimensions and position and is aligned at all frequencies, then it is an easy matter to ensure that this point is co-located with the input focal plane of the reflector system. Assuming constancy in the beamwaist, the reflector system will Fourier transform the aperture field at the input focal plane into an aperture field at the output focal plane. The distribution at this output focal plane will be frequency dependent but the antenna farfield radiation pattern is the Fourier transform of this aperture distribution. The farfield angular antenna pattern is independent of frequency. The desired result of constancy in the output farfield radiation pattern from the reflector has therefore been obtained by a double Fourier transform of the input from the quasi-optics because a double Fourier transform is not frequency dependent. This approach to the analysis of the entire antenna system is a first order alternative to a more complex approach using amplitude distributions and phase centres throughout the analysis. However, if the quasi-optics output beam waist varies in position, then the reflector system will behave as though the focal feed moves. This will result in degraded farfield patterns. The movement can be in two directions, axially along the reflector center ray and at right angles to this. The axial movement is more important as it controls the design of the optical chain.

A. Axial Movement

Under normal circumstances, defocussing a reflector by a small amount is not regarded as important [7]. The gain decreases slightly and the beam widens, resulting in little change in beam efficiency as defined in (1). This assumes that the increase in footprint size is acceptable. On the other hand, for the multi-channel systems considered here, there are generally stringent requirements on the footprint match between frequency channels. The result is an unusually tight specification on the final beam waist position at all frequencies in all channels. This would not be so if higher order modes were absent. It is the relative slippage of the higher order modes at the true focus as the frequency changes which causes degradation.

B. Transverse Movement

Transverse motion tilts the final output beam direction through an angle which is dependent on the geometry of the reflector and the direction of the motion. On the whole, translational movements are likely to be small and a matter of mechanical adjustment of all the optical components.

C. Aberrations

A detailed knowledge of the complex field (amplitude and phase) which falls on the various components of the reflector system may be necessary in studying all possible system abberations. A phase center based on Gaussian beam mode analysis has been found [8] to be preferable for this purpose to the use of standard definitions of "phase center" which depend on the angular extent of the beam intercepted by the elements of the reflector system and on the distance of interception.

V. Shape of the Output Radiation Pattern

When the phase slippage between the modes from a corrugated horn is changed systematically, the output radiation patterns of the horn are also changed. Fig. 2 shows samples with a beam waist of 3.05 wavelengths. Clearly this will have an effect on the farfield radiation pattern of the associated reflector system. This will lead to changes in farfield beamwidth and beam efficiency since the taper at the edge of the reflector is varying. Such changes in effective feed radiation pattern will alter the beamwidth and the beam efficiency. Figs. 3 and 4 are for a Gaussian mode set illuminating a reflector of output aperture 220.0 mm with an offset angle of 90.0 degrees and a focal length of 135.0 mm at a frequency of 89 GHz. The resulting beam efficiency will not be acceptable for a specified lower limit of 95% since no allowance has been included for misalignments, system losses and stray radiation in

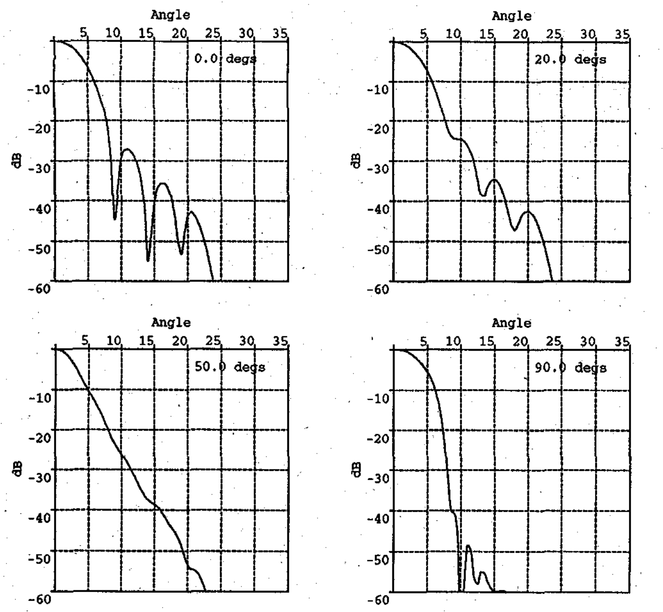

Fig. 2. Farfield radiation patterns from a corrugated horn. Fundamental mode slippage is parameter.

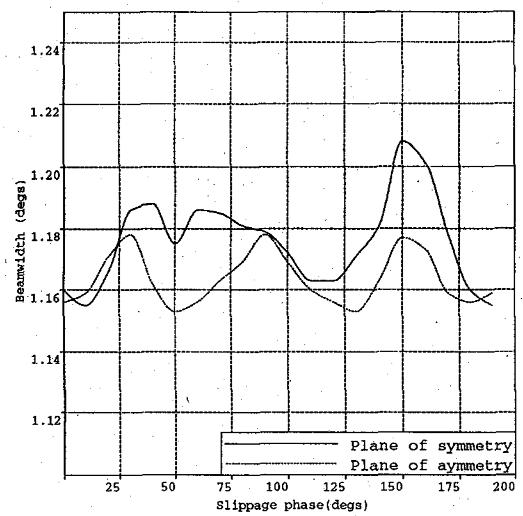

Fig. 3. Effect of fundamental mode slippage on beamwidth at 89 GHz.

the form of diffracted energy. These computations were carried out using a Gaussian beam-mode feed package integrated with GRASP [9].

To avoid the variations due to relative phase slippage within a frequency band, a quasi-optics system must have optics independent of frequency. This is achieved by using a zero flare angle horn or, more usefully, a profiled horn with a zero flare angle at the aperture, and putting optical components, such as mirrors and lenses, at distances apart equal to the sum of their focal lengths. This ensures that there are no relative mode slippages between alternate beamwaists within the quasi-optic chain. The relative slippage from one mode to the next between successive beam waists is π radians and two such changes return the set of modes to its original condition.

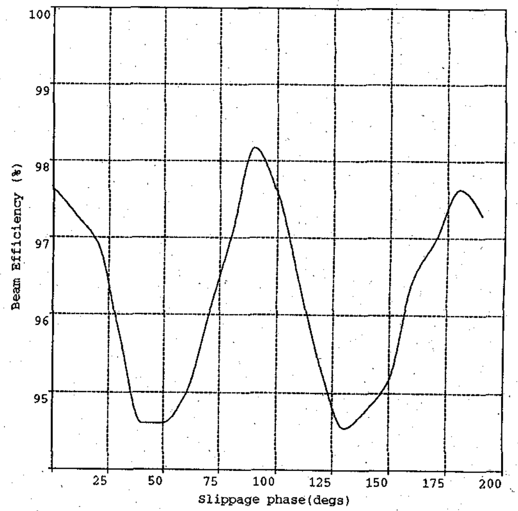

Fig. 4. Effect of fundamental mode slippage on antenna beam efficiency at 89 GHz.

VI. Conclusion

There are two possibilities for maintaining constant performance over a wide frequency range from a reflector antenna fed from a quasi-optical feed.

One possibility (the preferred route) requires that the quasi-optics system itself be frequency independent. Then the presence of higher order modes in the field will not alter the output feed radiation pattern at the reflector focus because there will be no inter-modal phase shifts.

If this is not feasible, the power in the higher order modes must be reduced as much as possible and consideration should be given to careful truncation of the beam to reduce the amplitude of contaminating higher order modes. It must be accepted that these can not be fully removed and will generate changes which will be apparent as footprint changes across the band as the modes slip relative to one another. The aim must be make the beams through the quasi-optics system as pure as possible.

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Patricia R. Foster (M'85) received the B.Sc. in physics from Edinburgh University and the Ph.D. degree from Cambridge University in 1956 and 1960, respectively.

From 1961 to 1963, she worked at the Royal Radar Establishment and lectured at Aberdeen University in Physics from 1966 to 1971. After working at Marconi Research Centre, Baddow and ERA Technology Leatherhead, she was Manager, New Antenna Systems, at British Aerospace, Filton until 1984 when she formed her own company. Dr. Foster's career has been in the antenna field and recent interests have been in quasi-optics for multiband use for civil and military applications. She is particularly interested in the application of computational electromagnetics to antenna problems. She has been Chairman of the IEE International Conference on Antenna and Propagation (1991) and is currently Chairman of the 8th Annual Symposium on the Applied Computational Electromagnetics for 1992.

Richard J. Wylde was born in London on February 16, 1958. He was educated at Eton and Sidney Sussex College, Cambridge from where he received the B.A. degree in Natural Sciences in 1979.

He joined Prof. Derek Martin's Engineering Physics Group in Queen Mary College (Now QMW), University of London, receiving the Ph.D. degree in Physics in 1985. Since then he has has split his time between academic work as a Visiting Research Fellow at QMW and running both Thomas Keating Ltd., Precision Engineers, and QMC Instruments Ltd., manufacturers of cooled InSb detectors. Dr. Wylde's research interests lie in the design of MM-Wave and FIR Optics for use in Plasma Diagnostics, Astronomy and Remote Sensing.